LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2024**

PMT1MC02 – REAL ANALYSIS-I

Date: 11-11-2024 Dept. No. Max. : 100 Marks

Time: 01:00 pm-04:00 pm

	SECTION A – K1 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
1	**Answer the following**
a)	Are the limit and the limit point same? Justify your answer.
b)	State mean value theorem.
c)	What distinguishes the Riemann Stieltjes integral from Riemann integral?
d)	Define uniform convergence of sequences.
e)	What is your understanding about Weirstrass's approximation theorem in real analysis?
	SECTION A – K2 (CO1)
	Answer ALL the questions **(5 x 1 = 5)**
2	**MCQ**
a)	In a metric space (M, ρ), an open sphere of radius r about a, $S(a, r) = \cdots$ (i) $\{x \in M : \rho(x, a) \leq r\}$ (ii) $\{x \in M : \rho(x, a) \neq r\}$ (iii) $\{x \in M : \rho(x, a) > r\}$ (iv) $\{x \in M : \rho(x, a) < r\}$
b)	If f has derivative at c and g has derivative at f(c) then gof has a ....at c. (i) Compact (ii) Complete (iii) Connectedness (iv) Derivative
c)	Upper Riemann Stieltjes integral of f with respect to $\propto$ over $[a, b]$ is.......... (i) $\sup L(P, f, \propto)$ (ii) $\inf L(P, f, \propto)$ (iii) $\sup U(P, f, \propto)$ (iv) $\inf U(P, f, \propto)$
d)	If a sequence of real numbers is convergent then (i) it has two limits (ii) it is bounded (iii) it is bounded above but may not be bounded below (iv) it is bounded below but may not be bounded above
e)	Every equicontinuous family on a .... is uniformly bounded (i) closed set (ii) derived set (iii) compact set (iv) none of the above

SECTION B – K3 (CO2)	
	Answer any THREE of the following **(3 x 10 = 30)**
3	Show that a mapping f of a metric space X into a metric space Y is continuous iff $f^{-1}(V)$ is closed in X for every closed set V in Y.
4	If f is a real differentiable function on $[a,b]$ and suppose $f'(a)<\lambda<f'(b)$.defined on $[a,b]$. Show that there is a point $x\epsilon(a,b)$ such that $f'(x)=\lambda$.
5	If f is monotonic on $[a,b]$ and α is monotonically increasing continuous function on $[a,b]$ then show that $f\in RS(\alpha)$ on $[a,b]$.
6	Interpret that the sum function of a uniformly convergent series of continuous function is itself continuous.
7	Show that there exists a real continuous function on the real line which is nowhere differentiable.
SECTION C – K4 (CO3)	
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Suppose f is continuous on $[a,b]$. $f'(x)$ exists at some point $x\in[a,b]$, g is defined on an interval I which contains the range of f and g is differentiable at $f(x)$. Determine that if $h(t)=g(f(t)), a\le t\le b$ then h is differentiable at x and $h'(x)=g'(f(x))f'(x)$.
9	If $f\in RS(\alpha)$ on $[a,b]$ and C is a constant function. Determine that $Cf\in RS(\alpha)$ on $[a,b]$ and $C\int_a^b fd\alpha=\int_a^b C\,fd\alpha$.
10	State and prove Cauchy criterion for uniform convergence.
11	Let α be monotonically increasing function on [a,b¿ and let $\{f_n\}$ be a sequence of real valued functions defined on $[a,b]$.Such that $f_n\in RS(\alpha)$ on $[a,b]$ for $n=1,2,3\ldots$If $f_n\to f$ uniformly on $[a,b]$, Then determine that f is itself integrable and $\int_a^b fd\alpha=\lim_{n\to\infty}\int_a^b f_n d\alpha$.
SECTION D – K5 (CO4)	
	Answer any ONE of the following **(1 x 15 = 15)**
12	a) Defend that every neighbourhood is an open set. b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then criticize that $f(x)$ is compact. **(5+10)**
13	a) Suppose f and g are defined on $[a,b]$ and are differentiable at a point $x\in[a,b]$ then $f+g$, $f.g$, f/g are differentiable at x, then support that (i) ¿ (ii) ¿ (iii) ¿ b) Let $f(x)=\begin{cases}x^2, x\neq1\\0, x=1\end{cases}$ determine that $\lim_{x\to1}x^2$ if limit exists. **(12+3)**
SECTION E – K6 (CO5)	
	Answer any ONE of the following **(1 x 20 = 20)**
14	a) If $P^{¿}$ is the refinement of the partition P then discuss that $L(P,f,\alpha)\le L(P^{¿},f,\alpha)$ and U$(P^{¿},f,\alpha)\le L(P,f,\alpha)$.

	b) Let $f(x)=x$ and $\alpha(x)=x^2$. Does $\int_0^1 f d\alpha$ exists? If it exists then find its value. **(15+5)**
15	Discuss and justify whether a uniformly continuous polynomial P_n is real for a continuous complex function f in $[a,b]$.

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